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An experimental-based model for prediction of flow noise with drag reducing polymers

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Abstract

The drag reduction characteristics of certain high molecular weight polymers have been studied by various investigators. Because of the polymer's ability to reduce turbulent shear stress and dependence of the boundary layer wall pressure spectral amplitude on the shear stress, polymer has the potential to suppress noise and vibration caused by the boundary layer unsteady pressures. Compared to its effect on drag reduction, polymer additive effects on turbulent boundary layer (TBL) wall pressure fluctuations have received little attention. Kadykov and Lyamshev [Sov. Phys. Acoust. 16 (1970) 59], Greshilor et al. [Sov. Phys. Acoust. 21 (1975) 247] showed that drag reducing polymer additives do indeed reduce wall pressure fluctuations, but they have not established any scaling relationship which effectively collapse data. Some effort has been made by Timothy et al. [JASA 108 (1) (2000) 71] at Penn State University to develop a scaling relationship for TBL wall pressure fluctuations that are modified by adding drag reducing polymer to pure water flow. This paper presents a theoretical model based on the work of the Timothy et al. team at ARL, Penn State University. Through this model one can estimate, reduction in TBL flow induced noise and vibration for rigid smooth surfaces due to release of drag reducing polymers in boundary layer region. Using this theoretical model, flow noise as experienced by a typical flush mounted hydrophone has been estimated for a smooth wall plate as a function of polymer additive concentration. Effect of non-dimensionalisation of the wall pressure fluctuations frequency spectra with traditional outer, inner and mixed flow variables will also be addressed in the paper. The paper also covers a model based on molecular relaxation time in polymer additives which not only reduce drag but also flow induced noise up to certain polymer concentration.

Keywords: Flow noise; TBL noise; Drag reduction; Polymer addition; Wall pressure fluctuations

1. Introduction

Numerous synthetic and natural polymeric materials have been found to give significant reductions in frictional drag under turbulent flow conditions since the discovery of the phenomenon by Toms in 1948. The noticeable features of these polymer solutions are very long chain structure with few side branches, high molecular weight etc. Solution of these additives invariably exhibit certain detectable elastic effects at

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extremely low concentration. Because of polymer's ability to reduce turbulent shear stress and the dependence of the turbulent boundary layer (TBL) wall pressure spectral amplitude on the shear stress, polymer has the potential to suppress noise and vibration caused by TBL unsteady pressures. Compared to its effect on drag reduction, however, polymer effects on TBL wall pressure fluctuations have received little attention.

Barker showed that drag reducing polymer additives indeed reduce wall pressure fluctuations, but was unable to effectively collapse his data. Work reported by Kadykov and Lyamshev (1970) and Greshilor et al. (1975) has also showed that polymer induced drag reduction is associated with a reduction in wall pressure fluctuations, but specific scaling relationships have remained elusive. A recent paper by Timothy et al. (2000) has developed a scaling relationship for TBL wall pressure fluctuations that are modified by adding drag reducing polymers to pure water flow. The objective of this paper is to develop a theoretical model for prediction of wall pressure fluctuations with drag reducing polymers based on the experimental data published by Timothy et al. (2000). The paper presents an empirical model for estimation of point power spectrum for different polymer concentrations of polyox solution. It is further extended, based on molecular relaxation time theory, into a generalized mathematical model for prediction of flow noise reduction for any drag reducing polymer species.

2. Theoretical analysis

Drag reduction in TBL is due, primarily, to the effect of the polymer additive on the fluid motion very near the wall. The presence of the additive outside of the viscous sublayer in turbulent flow have been shown by experiments to have little or no effect on the flow rate–shear stress relationship. The fact that drag reducing polymers are effective only within viscous-sublayer indicates that one should look for acoustic sources which lie within this region which approximately constitute 1–2% of total TBL.

2.1. Acoustic sources in viscous-sublayer of TBL

It may be seen from Eq. (1) given below that the primary source of sound in TBL is the fluctuating Reynolds stresses $\rho u_i' u_j'$ where $u_i' u_j'$ are velocity fluctuations occurring in an incompressible turbulent flow in x_i and x_j directions respectively (Hardin, 1991)

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x_1}\right)\rho' - a_0\nabla^2\rho' \cong \rho_0 \frac{\partial^2}{\partial x_i \partial x_j}(u_i' u_j') \tag{1}$$

The terms in the equation a_0, ρ', U are sound velocity, fluctuating density and flow velocity respectively. This equation is derived from Lighthill's acoustic analogy by changing the forcing term given on right hand side. Original Lighthill's expression is valid when the source is surrounded by a medium in uniform motion and a rigid boundary.

The fluctuating stresses within the TBL are mainly due to hydrodynamic instabilities such as formation of Tollemein–Schlichting (TS) waves, vortical phenomenon such as horse shoes, bursts, streaks, spots and their mutual interaction which ultimately manifests as sources of acoustic noise.

Literature surveys indicate that this viscous sublayer region of boundary layer is generally filled with counter rotating pairs of stream-wise vortices that lie between streaks discovered by Kline et al. (1967). The origin of these stream-wise vortices is not well understood although Taylor–Gotler instability is suspected. Intermittently the streaks begin to oscillate and then break up in fairly violent motions—called bursts. The bursts are thought to be produced as the passage of a vortex causes the local velocity profile to be instantaneously unstable, a phenomenon referred to as the called Kalvin–Helmholtz shear layer instability.

The streaks lifts up from the wall and bursts in a skewed helical vortex pattern. During this process, very large fluctuating Reynold's stresses and much small scale turbulence are produced in a highly localized area constituting the last 1% or so of the TBL closest to the wall. Measurements of wall pressure fluctuations carried out on an axisymmetric body by Timothy et al. (2000) have also shown that inner flow variables scaling provides more effective collapse of data which also supports the arguments that noise reduction following the polymer addition takes place in the viscous-sublayer region. This supports the view that if drag reduction mechanism is taking place in viscous sublayer, then reduction in Reynolds stresses will not only reduce drag but also wall pressure fluctuation as is evident from RHS of Eq. (1).

Since it is difficult to solve Eq. (1), recourse is always taken to modeling of the experimental data to estimate the forcing function which is essentially statistical in nature. The succeeding paragraphs discuss these aspects.

2.2. Wall pressure spectrum with polymer additive

Modeling of the wall pressure spectrum needs essentially two important parameters viz. (i) Point Power Spectrum, $P(\xi,\omega)$ as $\xi \to 0$, which describes the frequency distribution of fluctuating energy (ii) Cross Power Spectrum which describes spatial distribution of fluctuating energy. These two parameters are required for the solution of several applied problems pertaining to the generation of vibrations by TBL flow in various engineering structures and subsequent re-radiation of sound into the environment.

There are many models of point power spectrum for normal fluid flows used by various investigators in the last several decades, but none of these models are useful when polymers are added beneath the TBL (Bhujanga Rao, 1995). The author of this paper has therefore taken data published by Timothy et al. and empirically derived an expression describing wall pressure spectra. For this purpose, the wall pressure spectra with drag reducing polymer additives measured by a finite size transducer and non-dimensionalised with the inner and outer variables providing the best collapse of the data was considered. This data was corrected for transducer size to arrive at point power spectrum of wall pressure fluctuations.

2.2.1. Correction for transducer finite size

It is well known fact that experimental resolution of the structure of turbulence is limited by the finite size of the measuring devices. Simply stated, they can not resolve fluctuations having wave lengths smaller than the transducer dimension. This fact is of considerable importance in the measurement of pressures under a TBL. As a consequence, all measurements which have the condition that $\frac{\omega R}{U} > \frac{\pi}{2}$ are subject to considerable error. The basic theory for making appropriate corrections for this effect depends upon the a priori knowledge of the true pressure field. In as much as this field is precisely the unknown function being sought, a very piquant situation occurs. To alleviate this difficulty, a correction factor has been introduced in all the data measured by Timothy et al. The correction factor used in this paper is due to White (1967) and is given by

$$\frac{\phi_{\mathrm{m}}(\omega)}{\phi(\omega)} = \sum_{j=1}^{N} \left[\frac{\sin\left(\frac{\omega l_j}{2u_{\mathrm{c}}}\right)}{\frac{N\omega l_j}{2u_{\mathrm{c}}}} \right]^2 \tag{2}$$

Here the circular transducer of 2.54 mm was divided into N number of strips with length l_j for the jth strip. u_{c} is the convective velocity of turbulent field. $\phi_{\mathrm{m}}(\omega)$ and $\phi(\omega)$ are measured and corrected spectral density of pressure which is applicable for circular transducer. The wall pressure spectra before and after correction for the transducer size is given in Fig. 1.

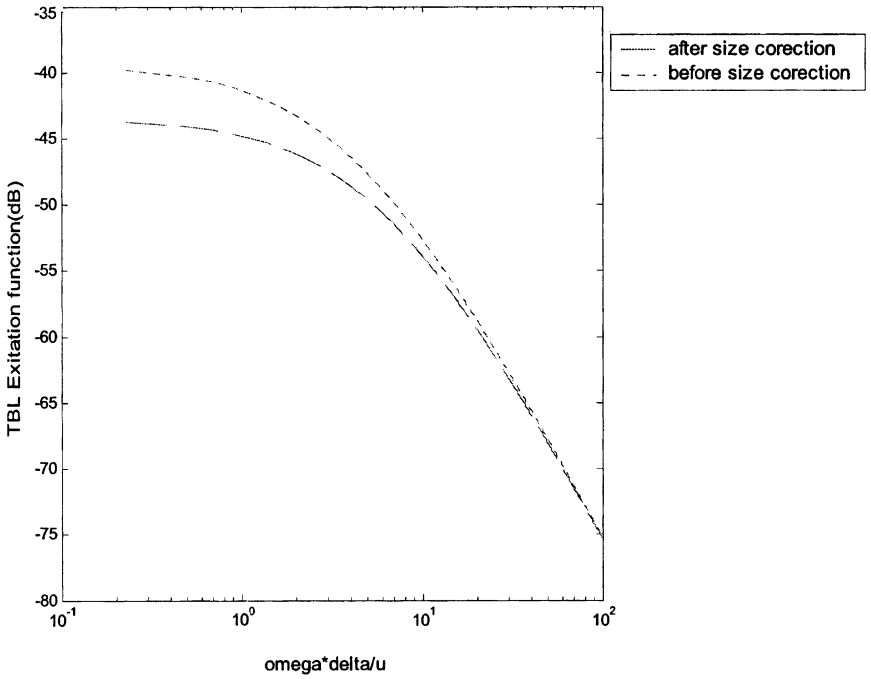

Fig. 1. Correction for transducer size.

2.2.2. Point power spectrum

The non-dimensionalised spectrum, after correcting for the transducer size, has been empirically modeled and the following expression for point power spectrum $P(0,\omega)$ has been derived.

$$P(0,\omega) = \frac{\rho U^2 \tau \delta^*}{\left[u\left(2 + 0.47\left(\frac{\omega\delta^*}{u}\right)^{1.16}\right)^4\right]} \tag{3}$$

U free stream velocity
τ wall shear stress for different values of polymer concentration
u friction velocity for different values of polymer concentration
δ^* boundary layer displacement thickness for different values of polymer concentration

Eq. (3) is only applicable to the case of a particular polymer which in the present case is polyox (polyethelene oxide) solution.

From Table 1, it may be observed that $\frac{\delta^*}{u}$ is constant equal to 2.26×10^{-3} for all concentrations of the polymer.

Substituting the constant value for $\frac{\delta^*}{u}$, Eq. (3) further simplifies to

$$P(0,\omega) = \frac{\rho U^2 \tau (2.26 \times 10^{-3})}{[(2 + 4.0 \times 10^{-4}(\omega)^{1.16})]^4} \tag{4}$$

2.2.3. Modelling of frequency wave vector spectrum

Assuming the turbulent fields as statistically stationary and homogeneous, a two point moment such as space–time correlation function $R(\xi,t)$ can be used to describe the field.

Table 1

Polymer concentration (WPPM)	$\frac{\delta^*}{u}\times 10^{-3}$	Drag reduction	Reduction in wall pressure fluctuations at 500 Hz	Relaxation time (s)
0	2.03	0	–	2×10^{-3}
1	2.44	32%	1 dB	4×10^{-3}
5	2.30	50%	3	8×10^{-3}
10	2.18	58%	4	1×10^{-2}
20	2.34	72%	6	2.5×10^{-2}

Note: solvent: water, polymer: polyox solution.
Average value of $\frac{\delta^*}{u} = 2.26\times10^{-3}$.
Columns 2 and 3 are from Timothy et al. paper.

$$P(\xi,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R(\xi,\tau)^{-\mathrm{i}\omega\tau}\,\mathrm{d}\tau \tag{5}$$

Then the frequency-wave vector spectrum is given by

$$E(k,\omega) = \frac{1}{(2\pi)^3}\int\int_{-\infty}^{\infty}\int R(\xi,\tau)^{-\mathrm{i}(k\cdot\xi+\omega\tau)}\,\mathrm{d}\xi\,\mathrm{d}\tau = \frac{1}{(2\pi)^2}\int\int_{-\infty}^{\infty} p(\xi,\omega)\mathrm{e}^{-\mathrm{i}k\cdot\xi}\,\mathrm{d}\xi \tag{6}$$

$\xi = (\xi_1,\xi_2)$ is the vector of spatial separation between points of the field in the plane of the wall. k is the wave vector with components k_1 and k_2 corresponding to ξ_1 and ξ_2. Abundant data is available on the dimensionless cross spectrum $\gamma(\xi,\omega) = P(\xi,\omega)/P(\omega)$ for which measurement techniques have long been established. Measurements of the frequency-wave vector spectrum $E(k,\omega)$ have begun in comparatively recent times, but the compilation and systematization of this new experimental information has led to one of the latest turbulent field models given by Smol'yakov and Tkachenko (1991). The construction of this model was preceded by detailed measurements of the dimensionless cross spectra of wall pressure fluctuations in gradient free turbulent boundary layers. But there are no models available in published literature for the case of turbulent flow fields in the presence of drag reducing polymer additive in the viscous-sublayer. But analysis by the present author indicate that the wall pressure model of Smol'yakov and Tkachenko describes all characteristics of the behaviour of the dimensionless longitudinal and transverse cross spectra by means of generalized decay rates $A_1(\omega)$ and $A_2(\omega)$ as given below.

$$|\gamma(\xi_1,0,\omega)| = \exp\left[-A_1(\omega)\left|\frac{\omega\xi_1}{u_\mathrm{c}}\right|\right] \tag{7}$$

$$|\gamma(0,\xi_2,\omega)| = \exp\left[-A_2(\omega)\left|\frac{\omega\xi_2}{u_\mathrm{c}}\right|\right] \tag{8}$$

Here

$$A_1 = \alpha_1\varphi(\bar\mu/\bar\omega),\quad A_2 = \alpha_2\varphi(\bar\mu/\bar\omega)$$
$$\varphi(\bar\mu/\bar\omega) = [1-\bar\mu/\bar\omega+(\bar\mu/\bar\omega)^2]^{1/2}$$
$$\bar\mu = \mu_1\bar u_\mathrm{c}/\alpha_1 = \mu_2\bar u_\mathrm{c}/\alpha_1,\quad \bar\omega = \omega\delta/U,\quad \bar u_\mathrm{c} = u_\mathrm{c}/U$$

The values of the constants for agreement with the experimental data are $\alpha_1 = 0.124$, $\alpha_2 = 0.8$, $\mu_1 = 0.031$, $\mu_2 = 0.20$, and $\bar u_\mathrm{c} = 0.8$, so that $\bar\mu = 0.2$. To facilitate the ensuring discussion, we introduce $m_0 = \alpha_2/\alpha_1 = \mu_2/\mu_1 = 6.45$ and denote $A_1(\omega) = A$ and $A_2(\omega) = m_0A$.

The modulus of the dimensionless cross spectrum is given by

$$|\gamma_0(\xi_1,\xi_2,\omega)| = \exp\left\{ -\left[\left(A\frac{\omega\xi_1}{u_c}\right)^2 + \left(m_0 A\frac{\omega\xi_2}{u_c}\right)^2\right]^{1/2}\right\} \tag{9}$$

With Fourier transformation Eq. (9) takes the form

$$E(k,\omega) = A\frac{P(\omega)}{2\pi m_0}\left(\frac{U_c}{\omega}\right)^2 h(w)[F(k,w) - \Delta F(k,w)] \tag{10}$$

where k is the wave number which gives no of waves per unit length

$$F(k,\omega) = [A^2 + (1 - kU_{c1}/\omega)^2 + (k_2U_c/m_0\omega)^2]^{-3/2}) \tag{11}$$

$$\Delta F(k,\omega) = \frac{1}{n}\left\{1 + A^2 + \frac{n}{m_1}\left[\left[m_1 - \frac{k_1u_c}{\omega}\right]^2 + \left[\frac{k_2U_c}{\omega}\right] - m_1^2\right]\right\}^{-3/2} \tag{12}$$

$$h(w) = \left[1 - \frac{m_1A}{m_0n^2\sqrt{G}}\right]^{-1} \tag{13}$$

$$G = 1 + A^2 - m_1n \tag{14}$$

$$P(\xi,\omega) = p(\omega)h(\omega)[|\gamma_0(\xi,w)|e^{t(w\xi,/u_c)} - \Delta\gamma(\xi,w)] \tag{15}$$

Subtitling in Eq. (4) in Eq. (10) give the frequency wave vector spectrum required for estimation of response of a structure or a sensor to TBL excitation with polymer additives.

3. Response of square hydrophone due to TBL excitation with polymer additives

The response of a square hydrophone subjected to TBL wall pressure fluctuations with and without polymer additives has been estimated as a typical example by evaluating the following integral

$$Q(\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E(k,\omega)H(k,\omega)\,dk_1\,dk_2 \tag{16}$$

where $H(k,\omega)$ is the hydrophone function given by

$$\left(\frac{\sin(k_1l_x/2)}{k_1l_x/2}\frac{\sin(k_2l_y/2)}{k_2l_y/2}\right)^2 \tag{17}$$

where l_x and l_y are the length of the sides of hydrophone.

The results of the hydrophone response for polyox solution for concentration varying from 1 to 20 wppm are presented in Fig. 5.

4. Generalisation of flow noise estimation for any drag reducing polymer

The point power spectrum in Eq. (4) which gives frequency distribution of turbulent pressure field is derived from data obtained with polyox solution. At the present time, there is no single theory of flow noise which fits all experimental evidence with different polymer additives in different situations. If it were possible to observe the configuration and behaviour of a macromolecule within a solution undergoing turbulent shear flow then we would be in a far better position to offer explanations for flow noise reduction mechanism.

Millward and Lilley (1974) while explaining drag reduction mechanism postulate that energy being transferred down the cascade from large to small eddies is partly transferred to the polymer molecules towards the high frequency end of the spectrum and this could alter the whole energy balance of the turbulence.

It has been observed by Virk et al. (1967) and others that within the region of drag reduction (which is same region for flow noise reduction also), the polymer relaxation times are equal or greater than the time scales of the smallest eddies. For example, Virk quotes a relaxation time for polyox WSR 301 as 0.0186 s. (This is within the range of value given in Table 1 by the present author). It is thus possible for molecule to distort and therefore absorb or dissipate energy from the small eddies. One can derive an approximate expression for mean square wall shear stress in terms of relaxation time (λ) well within the asymptotic region

$$\bar{\tau}^2 = \frac{\rho\mu U^2\omega}{2[1+\omega^2\lambda^2]^{1/2}} \tag{18}$$

For $\omega\lambda \gg 1$, it simplifies to

$$\bar{\tau}^2 = \frac{\rho\mu U}{2\lambda} \tag{19}$$

where μ is the fluid viscosity.

If λ is known for the additive polymer, one can estimate $\bar{\tau}^2$ and substitute in the following expression obtaining point power spectrum $P(0,\omega)$ given by Thachenko and Marshov (1989).

$$P(\omega) = \frac{\tau_w^2\delta^*}{U}\,\frac{5.1}{1+0.44(\omega\delta^*/U)^{7/3}} \tag{20}$$

Though measurement of relaxation time is very difficult, however, published literature provides data on this parameter to some extent. Substituting Eq. (19) in Eq. (10), one can obtain frequency wave vector spectra for any concentration of polymer.

Since λ increases with concentration upto a certain value, one can even estimate the asymptotic values beyond which further increase in concentration will not have any effect on flow noise reduction.

5. Results and discussion

(a) Eqs. (4), (10) and (19) together forms the basis for obtaining TBL excited spectrum in the presence of Polymer additive. These expressions can be used to estimate the acoustic response of a hydrophone array or vibration of a structure excited by turbulence in many hydroacoustic applications. The results obtained on TBL excitation forces are shown in Figs. 2–4 for three different frequencies. There is marginal reduction in excitation forces with the addition of polymer compared to pure water. The analysis is applicable for the case of polyox solution only. A generalized expression for estimating power spectrum for any polymer addition is given in Eq. (4) in terms of polymer relaxation time among others.

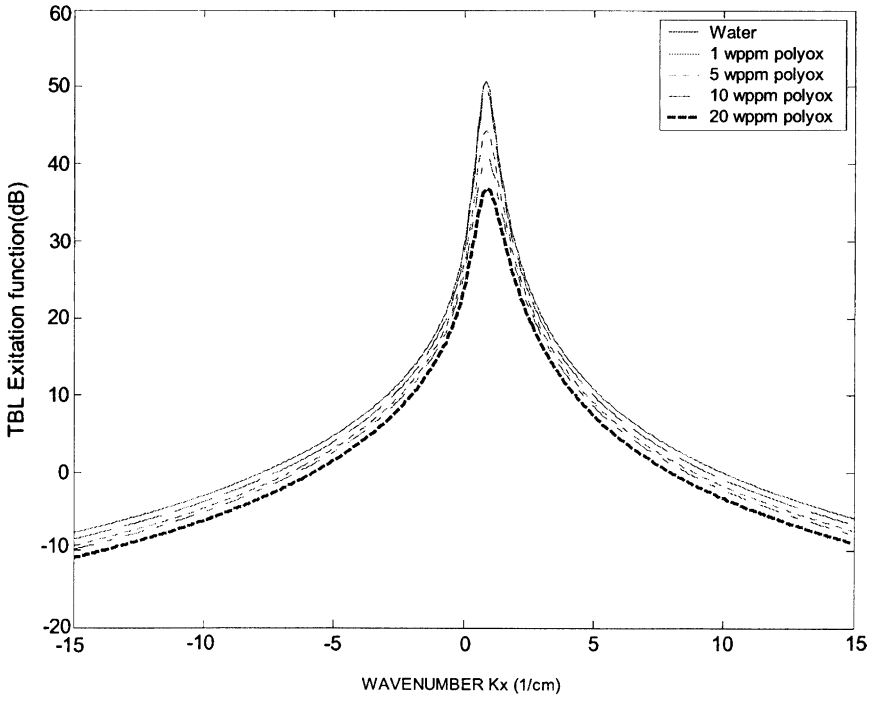

Fig. 2. TBL wave vector for 100 Hz with polymer concentration increasing.

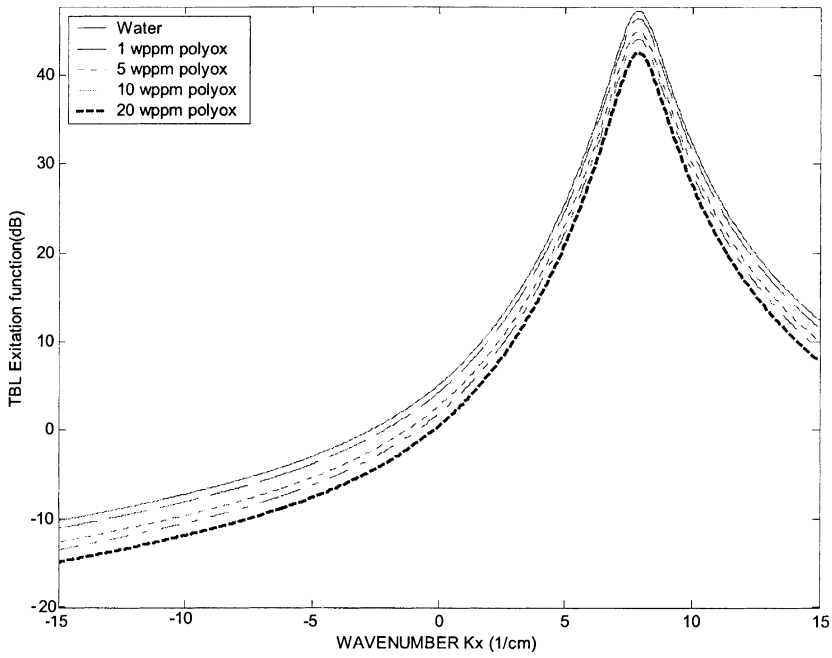

Fig. 3. TBL wave vector for 1000 Hz with polymer concentration increasing.

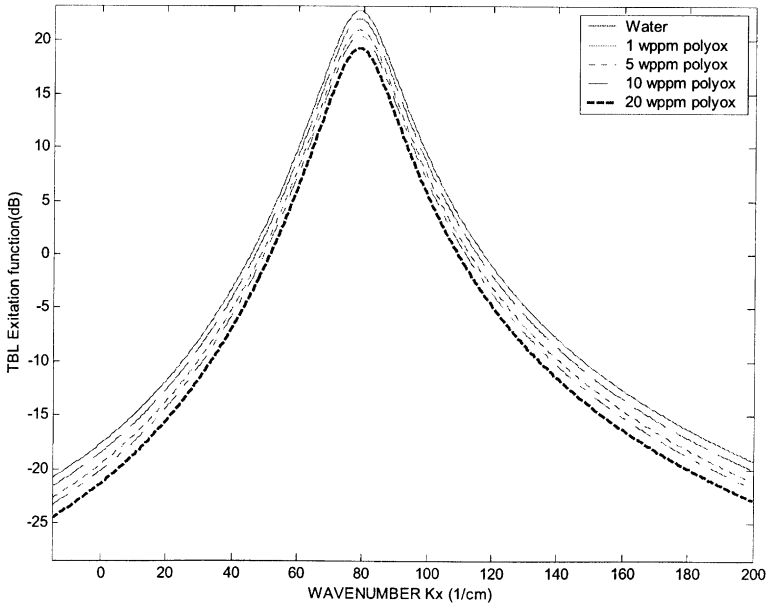

Fig. 4. TBL wave vector for 10 kHz with polymer concentration increasing.

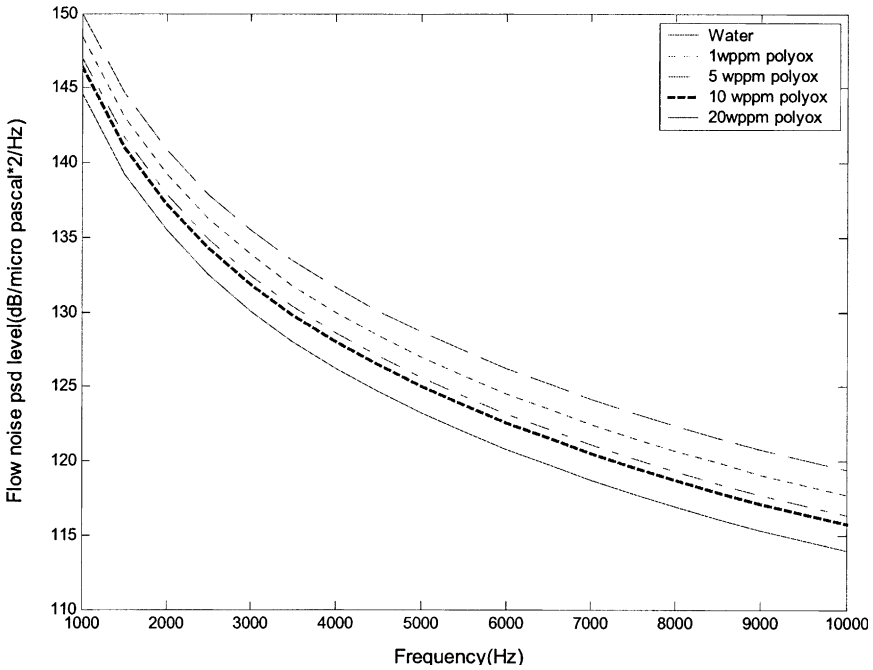

Fig. 5. Computed flow noise level with drag reducing polymer as perceived by square hydrophone size (2.5 cm).

The relaxation time estimated by the author for polyox solution compares well with the published literature.

(b) The point power spectrum given by Eq. (4) indicates that it is directly proportional to wall shear stress. Drag reducing polymers are known to give drag reduction due to reduction in the wall shear stress at the wall. Same reason is to be attributed for reduction in wall pressure fluctuations also. It may therefore be surmised that reduction in drag as well as flow noise are simultaneous phenomenon.

(c) Using the methodology given in Bhujanga Rao and Laxman (communicated, February 2002) flow noise levels as received by a square hydrophone of size 25 mm is shown in Fig. 5. Reduction of nearly 5–6 dB in flow noise level can be observed across a wide frequency range with polymer concentration varying from 1 to 20 wppm.

6. Summary

A empirical model for production of flow noise in the pressure of drag reducing polymer beneath the TBL has been presented. It is found that flow noise reduction can be obtained using drag reducing polymers to an extent of 5–6 dB in a typical case with polyox solution over a hydrophone array. Reduction in structural response excited due to TBL can also be excepted due to drag reducing polymers.

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